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Moduli Stabilization in Warped Compactifications at One Loop

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We review the moduli stabilization mechanism found in Garriga *et al.* (Garriga, J., Pujolas, O., and Tanaka, T. (2000). *Preprint* hep-th/0111277.) for a class of five-dimensional warped brane-world scenarios. Specifically, we consider solutions with a power-law warp factor and a bulk dilaton with logarithmic profile in terms of the proper distance in the extra dimension. This includes the Heterotic M-theory brane-world of Lukas *et al.* (Lukas, A., Ovrut, B. A., Stelle, K. S., and Waldram, D. (1999). *Physical Review D* **59**, 086001.) and Khoury *et al.* (Khoury, J., Ovrut, B. A., Steinhardt, P. J., and Turok, N. (2001). *Preprint* hep-th/0103239.) and the Randall–Sundrum (RS) model as a limiting case. In general, there are two moduli fields $y_{\pm}$, corresponding to the "positions" of two branes. Classically, the moduli are massless due to a scaling symmetry of the action. However, in the absence of supersymmetry, they develop an effective potential at one loop. Local terms proportional to some powers of the local curvature scale at the location of the corresponding brane are needed in order to remove the divergences in the effective potential. Such terms break the scaling symmetry and therefore act as stabilizers for the moduli. Moreover, for $q \gtrsim 10$, the observed hierarchy can be naturally generated by this potential, and the lightest modulus mass is of order $m_{-} \lesssim TeV$.

KEY WORDS: casimir energy; brane world; warped compactifications.

1. INTRODUCTION

Spacetimes with extra dimensions and a number of branes have recently been considered in order to construct phenomenologically interesting models of Nature (Antoniadis *et al.*, 1998; Arkani-Hamed *et al.*, 1998, 1999; Randall and Sundrum, 1999). From the four-dimensional point of view, the parameters describing the possible shape of the higher dimensional geometry, such as the distances between branes in the generically correspond to four-dimensional scalar fields φ collectively called moduli. The moduli may be massless at the classical level but in the absence of supersymmetry they tend to develop an effective potential at one loop. This happens already in the simplest Kaluza–Klein (KK) compactification, and even

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if there are no branes (Appelquist and Chodos, 1983a,b). In four-dimensional language, a field χ that lives in the bulk can be split in an infinite tower of massive KK fields, labeled by a discrete index n. The masses of these KK excitations $m_n(\varphi)$ depend on the moduli φ. Since, e.g., in Minkowski spacetime, a massive scalar induces a potential proportional to $m_n(\varphi)^4$, a KK tower generates an effective potential $V(\varphi)$ at one loop.

In most brane-world compactifications, the self-gravity of brane and bulk matter content induce a warp in the extra dimension. In this situation divergences proportional to world-volume operators on the brane are generated, so the subtraction of infinities in $V(\varphi)$ is not as simple as it is in flat space. As we will see, this gives rise to new interesting effects.

The effective potential in the Randall–Sundrum model (RS) (Randall and Sundrum, 1999) was computed in reference (Garriga *et al.*, 2000) using zeta function regularization. A somewhat unexpected feature of the result is the absence of logarithmic terms in the effective potential, which in turn results in a very small correction for the mass of the radion in the limit of large hierarchy. Several papers have recently appeared in the literature (Brevik *et al.*, 2000; Flachi and Toms, 2001; Goldberger and Rothstein, 2000; Nojiri *et al.*, 2000; Toms, 2000) where the effective potential is obtained in dimensional regularization through certain subtractions corresponding to renormalization of the brane tensions in the dimensionally extended spacetime. The agreement between the results indicates that both procedures are in fact equivalent. However, the RS model is specially simple in that it is built from AdS space (which is maximally symmetric). So all counterterms that one can construct from the geometry are proportional to the volume of the bulk or to the "area" of the branes.

Here we consider a class of warped brane-world models with a nontrivial bulk scalar and a power-law warp factor as the background-field configuration, in which the bulk is no longer AdS. In Garriga *et al.* (2001), the equivalence between dimensional and zeta function regularization for these models was shown for the computation of the effective potential $V(\varphi)$. In what follows, I shall review the computation of $V(\varphi)$ in these cases. We shall note that the class of background configurations we consider includes the Heterotic M-theory brane-world of Lukas *et al.* (1999), which may perhaps be relevant for the recently proposed Ekpyrotic universe scenario (Khoury *et al.*, 2001) as well as the RS model as a limiting case. The result we find can be split into two parts: a nonlocal contribution, which we may identify as the Casimir energy generated by the bulk field χ, plus a number of induced local operators on the brane, which in turn depend on the local curvature at the place where the brane sits. These terms are needed in order to absorb the divergencies in the potential. They are absent in flat spacetime and, although present, they are irrelevent in the RS model. However, in the cases considered here, they provide a natural mechanism for stabilizing the moduli, giving them sizeable masses.

The plan of the paper is the following. In Section 2 we present the background model. In Section 3 we discuss some aspects of the regularization procedure and in Section 4 we explicitly compute $V(\varphi)$ induced by nonminimally coupled fields using dimensional regularization. The origin of the (gravitational) hierarchy is reviewed in Section 5, and the mechanism for stabilizing the moduli is outlined in Section 6. Our conclusions are summarized in Section 7. The following discussion is based on recent work in collaboration with Garriga *et al.* (2001).

2. THE MODEL

Let us consider a five-dimensional system composed of a scalar field ϕ coupled to gravity. The fifth dimension is compactified on a Z_2 orbifold with two branes at the fixed points of the Z_2 symmetry. The scalar field potential takes an exponential form in the five-dimensional bulk, with similar terms localized on the brane. The action for the background fields is given by

$$S_b = \frac{-1}{\kappa_5} \int d^5x \sqrt{-g} \left(\mathcal{R} + \frac{1}{2}(\partial\phi)^2 + \Lambda e^{c\phi} \right) - \sigma_+ \int d^4x \sqrt{-g_+}\, e^{c\phi/2} - \sigma_- \int d^4x \sqrt{-g_-}\, e^{c\phi/2}, \tag{2.1}$$

where $\mathcal{R}$ is the curvature scalar, $\kappa_5 = 16\pi G_5$, where G_5 is the five-dimensional gravitational coupling constant. We have denoted the induced metrics on the positive and negative tension branes by $g^{+}_{\mu\nu}$ and $g^{-}_{\mu\nu}$, respectively. To find a solution of the equations of motion, we make an ansatz where the four-dimensional metric is flat,

$$ds^2 = dy^2 + a^2(y)\eta_{\mu\nu}\, dx^\mu\, dx^\nu, \tag{2.2}$$

with a x^μ-independent scalar field $\phi = \phi_0(y)$. The positive and negative branes are placed at $y = y_+$ and y_-, respectively. Under these assumptions, the equations of motion for (a, ϕ) in the bulk become

$$\left(\frac{\dot a}{a}\right)^2 = \frac{1}{12}\left(\frac{1}{2}\dot\phi^2 - U(\phi)\right),$$
$$\ddot\phi + 4\,\frac{\dot a}{a}\,\dot\phi = U'(\phi), \tag{2.3}$$

where $U(\phi) \equiv \Lambda e^{c\phi}$, a dot represents differentiation with respect to y and a prime represents differentiation with respect to ϕ.

As shown in Youm (2000, 2001), there is a solution of Eq. (2.3) for any value of c given by

$$\phi_0 = -\sqrt{6q}\, \ln(y/y_0),$$

$$a(y) = (y/y_0)^q, \tag{2.4}$$

where

$$q = \frac{2}{3c^2}, \quad y_0 = \sqrt{\frac{3q(1-4q)}{\Lambda}}. \tag{2.5}$$

(Constant rescalings of the warp factor are of course allowed, but unless otherwise stated, we shall take the convention that $a(y) = 1$ at $y = y_0$.) Assuming $y_- < y_+$, the boundary conditions that follow from Z_2 symmetry imposed on both branes are given by

$$\dot{\phi}_\pm = \mp\frac{c}{4}\sigma_\pm\, e^{(c/2)\phi_\pm}, \tag{2.6}$$

$$6\left.\frac{\dot{a}}{a}\right|_\pm = \pm\frac{1}{2}\kappa_5\sigma_\pm\, e^{(c/2)\phi_\pm}, \tag{2.7}$$

and they are satisfied if $\sigma_\pm$ are tuned to

$$\sigma_\pm = \pm\frac{1}{\kappa_5}\sqrt{\frac{48q\Lambda}{1-4q}}. \tag{2.8}$$

For later reference, we define the conformal coordinates by

$$z \equiv \left|\int \frac{dy}{a(y)}\right| = \frac{y_0}{|1-q|}\left(\frac{y}{y_0}\right)^{1-q}, \tag{2.9}$$

with which the metric is

$$ds^2 = a^2(z)\big(dz^2 + \eta_{\mu\nu}\, dx^\mu\, dx^\nu\big), \quad a(z) = (z/z_0)^\beta, \tag{2.10}$$

where

$$\beta = \frac{q}{1-q}, \quad z_0 = \frac{y_0}{|1-q|}. \tag{2.11}$$

Here we should mention that the direction of increasing z does not coincide with the direction of increasing y when $q > 1$.

In the absence of the branes, the spacetime (2.4) contains a singularity at $y = 0$. Of course, since we are considering the range between y_- and y_+, this singularity does not cause any problem. Our spacetime consists of two copies of the slice comprised between y_- and y_+, which are glued together at the branes. Hence, the fifth dimension is topologically an S^1/Z_2 orbifold.

For $q = 1/6$, this solution is precisely the Heterotic M-theory model of Lukas *et al.* (1999). On the other hand, the RS model, where the bulk is AdS and there is no scalar field, corresponds to the case with $q \to \infty$.

For fixed value of the coupling c, the solution given above contains only two physically meaningful free parameters, which are the locations of the branes y_- and y_+. This leads to the existence of the corresponding moduli, which are massless scalar fields from the four-dimensional point of view. In addition to these moduli, the massless sector also contains the graviton zero mode. To account for it, we generalize our metric ansatz (2.2) by promoting $\eta_{\mu\nu}$ to an arbitrary four-dimensional metric:

$$ds^2 = dy^2 + a^2(y)\tilde{g}_{\mu\nu}(x)dx^\mu dx^\nu. \tag{2.12}$$

For constant values of the metric and moduli, we have a solution of the equations of motions whose action vanishes. Hence, only the terms that depend on derivatives of the metric or derivatives of the moduli will survive in the action after the five-dimensional integration.

The induced metric on the branes is of the form

$$g^{\pm}_{\mu\nu} = a^2_{\pm}\big[\tilde{g}_{\mu\nu} + a^{-2}_{\pm}\partial_\mu y_\pm \partial_\nu y_\pm\big].$$

Consequently, the induced kinetic terms for the moduli $y_\pm$ come from the brane tensions together with the Hawking–Gibbons boundary terms, and the classical action for the moduli can be put in the form (Garriga *et al.*, 2001)

$$S_{\text{kin}} = \frac{-1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\left\{(\varphi^2_+ - \varphi^2_-)\tilde{\mathcal{R}} - \frac{6q}{q+1/2}\big[(\tilde{\partial}\varphi_+)^2 - (\tilde{\partial}\varphi_-)^2\big]\right\}. \tag{2.13}$$

Here we have introduced

$$\varphi_\pm \equiv \left(\frac{y_\pm}{y_0}\right)^{q+1/2},$$

and the four-dimensional Newton's constant G given by

$$G = \left(q + \frac{1}{2}\right)\frac{G_5}{y_0}. \tag{2.14}$$

The modulus corresponding to the positive tension brane has a kinetic term with the "wrong" sign. However, this does not necessarily signal an instability, because it is written in a Brans–Dicke frame. One can show (Garriga *et al.*, 2001) that in the Einstein frame the two moduli have positive definite kinetic terms.

In the RS limit $q \to \infty$, one can show (Garriga *et al.*, 2001) that the kinetic term corresponding to the combination of the moduli given by $\varphi^2_+ - \varphi^2_-$ disappears. This is to be expected, because the bulk is the maximally symmetric AdS space, so only the relative position of the branes $y_+ - y_-$ is physically meaningful and necessarily one combination of the moduli can be gauged away (see also Barvinsky, 2001, for a recent discussion of this case).

To conclude this section, let us comment on the reason why the moduli should be massless at the classical level. Under the global transformation

$$g_{ab} \to T^2 g_{ab}, \tag{2.15}$$

$$\phi \to \phi - (2/c)\ln T, \tag{2.16}$$

the action (2.1) scales by a constant factor

$$S_b \to T^3 S_b.$$

Here g_{ab} is the metric appearing in the action (2.1). Acting on a solution with one brane, the transformation simply moves the brane to a different location. Hence, all brane locations are allowed, from which the masslessness of the moduli follows. However, this is just a global scaling symmetry that, in principle, is not expected to survive quantum corrections. By means of a conformal transformation (Garriga *et al.*, 2001), we may "change variables" to a new metric $g^{(s)}_{ab}$ that is invariant under the scaling symmetry

$$g^{(s)}_{ab} = e^{c\phi} g_{ab}. \tag{2.17}$$

In this form, the scaling is just due to the shift in ϕ.

3. EFFECTIVE POTENTIAL

In this section we set up the framework for computing the contribution to the one-loop effective potential from a scalar field propagating in the bulk with a generic mass term, which may include couplings to the curvature of spacetime as well as couplings to the background scalar field ϕ. The (Euclidean) action for this field is given by

$$S[\chi] = \frac{1}{2}\int d^D x \sqrt{-g}\, \chi P \chi, \tag{3.1}$$

where we have introduced the covariant operator

$$P = -(\Box_g + E).$$

Here $\Box_g$ is the d'Alembertian operator associated with the metric g_{ab}, and $E = E[g_{ab}, \phi]$ is a generic "mass" term. Typically, this takes the form $E = -m^2 - \xi \mathcal{R}_g$, where m is a constant mass, $\mathcal{R}_g$ is the curvature scalar, and ξ is an arbitrary coupling. However, in general E can also depend on any background fields, such as the dilaton ϕ. Throughout this section we shall leave E unspecified, although later on we shall restrict attention to the massless case $m^2 = 0$ for explicit calculations.

3.1. Ambiguity in the Definition of the Measure

The effective potential V per unit comoving volume parallel to the branes is defined by

$$e^{-\mathcal{A}V} = \int \mathcal{D}\chi\, e^{-S[\chi]} = (\det P)^{-1/2}, \tag{3.2}$$

where $\mathcal{A}$ is the comoving volume under consideration. Equivalently, we may write

$$V \equiv \frac{1}{2\mathcal{A}} \mathrm{Tr} \ln\left(\frac{P}{\mu^2}\right), \tag{3.3}$$

where the symbol Tr refers to the usual L_2 trace. For any operator $\mathcal{O}$, the trace can be represented as

$$\mathrm{Tr}[\mathcal{O}] = \sum_i \int d^D x\, g^{1/2} \Phi_i (\mathcal{O}\Phi_i) = \sum_i \int d^D x\, \tilde{g}^{1/2} \tilde{\Phi}_i (\mathcal{O}\tilde{\Phi}_i),$$

where Φ_i (or $\tilde{\Phi}_i$) form an orthonormal basis with respect to the measure associated with the metric g (or $\tilde{g}$, respectively). The definition of the trace is rather robust, in the sense that it is independent on the metric one uses in order to define the orthonormal basis, as long as the corresponding measures are in the same L_2 class. This will be the case, for instance, if the metrics are related by a conformal factor that is bounded above and below on the manifold.

Note that we have made a definite choice of the measure of integration in the path integral (3.2) when we have identified it with the trace of the logarithm of the covariant operator P. With this definition, the result is guaranteed to be covariant with respect to the metric g. That is, the result will only depend on geometric invariants constructed from the metric g and the scalar function E.[2]

However, Eq. (3.3), should not be regarded as the only possible definition of the measure of integration. As we discussed in Section 2, the action (2.1) is written in terms of the five-dimensional Einstein frame metric g_{ab}, but by using a conformal transformation that depends on the background scalar field one can construct a metric $g^{(s)}_{ab}$, which is invariant under the scaling symmetry. Classically, both metrics provide an equally valid description of spacetime, and in both cases we can write down generally covariant equations of motion in order to make identical predictions for physical quantities.

This arbitrariness in the choice of metric raises an ambiguity in the quantum theory because two metrics that differ by a conformal rescaling involving the scalar field will provide different measures of integration and different results for the effective potential. In (3.3) we chose covariance with respect to g_{ab}, but other choices are perfectly possible, or perhaps even preferable. In dimensional regularization, where the dimension of spacetime is extended to $5-\epsilon$, the ambiguity arises when we have to subtract divergences. The divergences are proportional to ϵ^{-1} times the integral of certain geometric invariants on the brane. The integral itself depends on ϵ, and the dependence is different for different choices of the "physical metric" out of which we construct the geometric invariants.

[2] Also, it may depend on additional scalar functions that are needed in order to specify the boundary conditions of the field on the brane. For simplicity here we shall restrict attention to the case of Dirichlet boundary conditions, where these functions are absent.

Since we have a classical scaling symmetry, a preferred choice for the measure of integration may be the one associated with the metric $g^{(s)}_{ab}$, which is invariant under scaling. However, even in this case the divergent part of the effective potential will not respect the scaling symmetry of the action, and consequently we need to introduce counterterms with the "wrong" scaling behavior. Hence, in what follows, we shall take the conservative attitude that the measure is determined in the context of a more fundamental theory (from which our five-dimensional effective action is derived), and we shall formally consider on equal footing all choices associated with metrics in the conformal class of g_{ab}, including of course $g^{(s)}_{ab}$.

3.2. Conformal Transformations and the KK Spectrum

Equation (3.2) relates the one-loop contribution induced by χ to the determinant of the operator P. However, as mentioned in the previous subsection, if we demand that the measure of integration be covariant with respect to $g^{(s)}_{ab}$, instead of g_{ab}, then the effective potential is given in terms of a different operator P_s, conformally related to P. The direct evaluation of the determinant of P, or P_s for that matter, turns out to be rather impractical, due to the complicated form of the implicit equation that defines their eigenvalues. For actual calculations it is convenient to work with a conformally related operator P_0 whose eigenvalues will be related to the KK masses.

Following Garriga *et al.* (2000), we introduce a one-parameter family of metrics that interpolate between our physical spacetime and a fictitious flat spacetime

$$g^{\theta}_{ab} = \Omega^2_{\theta}\, g_{ab}, \tag{3.4}$$

where θ parametrizes the path in the space of conformal factors. For $\theta = 0$ the fictitious metric g^{θ}_{ab} represents flat space, whereas for $\theta = 1$ it coincides with the physical metric (2.10). The actual path in the space of conformal factors will be unimportant, but for definiteness we shall take

$$\Omega_{\theta}(z) = \left(\frac{z}{z_0}\right)^{-\beta\,(1-\theta)}. \tag{3.5}$$

For $\theta = -1/\beta$, the metric g^{θ}_{ab} coincides with the metric $g^{(s)}_{ab}$ introduced in Section 2, which is invariant under the scaling transformation. As mentioned before, this metric corresponds to a five-dimensional AdS space, with curvature radius given by z_0.

Further, we define the operator P_{θ} associated with the metric g^{θ}_{ab} by

$$\Omega^{(D-2)/2}_{\theta} P_{\theta} \Omega^{(2-D)/2}_{\theta} = \Omega^{-2}_{\theta} P. \tag{3.6}$$

This operator can be written in covariant form as

$$P_{\theta} = -(\Box_{\theta} + E_{\theta}),$$

where

$$E_\theta = (D-2)\left(4\Omega_\theta^2\right)^{-1}\left[2\Omega_\theta^{-1}(\Box_g\Omega_\theta) + (D-4)g^{ab}(\partial_a \ln\Omega_\theta)(\partial_b \ln\Omega_\theta) + (4E/(D-2))\right],$$

and $\Box_\theta$ is the covariant d'Alembertian in the spacetime with metric g^θ_{ab}. Introducing $\chi_\theta \equiv \Omega_\theta^{(2-D)/2}\chi$, the action for the scalar field can be expressed as

$$S[\chi] = \frac{1}{2}\int d^Dx\sqrt{g_\theta}\,\chi_\theta\, P_\theta\, \chi_\theta. \tag{3.7}$$

Comparison with (3.1) and the discussion following (3.3) give the expression

$$V_\theta \equiv \frac{1}{2\mathcal{A}}\operatorname{Tr}\ln\left(\frac{P_\theta}{\mu^2}\right), \tag{3.8}$$

for the effective potential that is obtained from the covariant measure with respect to the metric g^θ_{ab}.

Of particular interest is $P_0 \equiv P_{\theta=0}$. This is the wave operator for the KK modes that one would use in a four-dimensional description. Moreover, it has the advantadge that its (Euclidean) eigenvalues $\lambda_{n,k} = k_\mu k^\mu + m_n^2$ separate as a sum of a four-dimensional part plus the KK masses m_n. In the following subsection, we shall discuss how $\det P_0$ is related to the determinant of our interest, $\det P$, or more generally to $\det P_\theta$ using dimensional regularization (we refer the reader to Garriga *et al.*, 2001, for a comparison between dimensional and zeta function regularizations).

3.3. Dimensional Regularization

A naive reduction to flat four-dimensional space suggests that the effective potential can be obtained as a sum over the KK tower:

$$V^D \equiv \mu^\epsilon \sum_n \frac{1}{2}\int \frac{d^{D-1}k}{(2\pi)^{D-1}}\log\left(\frac{k^2 + m_n^2(\varphi_i, D)}{\mu^2}\right). \tag{3.9}$$

Here $D = 4 + 1 - \epsilon$ is the dimension of spacetime, and we have added $(-\epsilon)$ dimensions parallel to the brane. The renormalized effective potential should then be given by an expression of the form

$$V(\varphi_i) = V^D - V^{\text{div}}, \tag{3.10}$$

and the question is what to use for the divergent subtraction V^{div}. Since Eq. (3.9) is similar to an ordinary effective potential in four-dimensional flat space,[3] one

[3] It should be mentioned also that each KK contribution in Eq. (3.9) is not just like a flat space contribution, because in warped compactifications the KK masses $m_n(\varphi, D)$ depend on the number of external dimensions parallel to the brane.

might imagine that V can be obtained from V^D just by dropping the pole term, proportional to $1/\epsilon$; but this is not true for warped compactifications. The point is that the theory is five-dimensional and the spacetime is curved, and this fact must be taken into account in the process of renormalization.

Rather than proceeding heuristically from (3.10), we shall take the definition of the effective potential equation (3.8) as our starting point, where it is understood that the formally divergent trace must be regularized and renormalized. In order to identify the divergent quantity to be subtracted, we shall use standard heat kernel expansion techniques. Let us introduce the dimensionally regularized expressions (Buchbinder *et al.*, 1992; Elizalde *et al.*, 1994; Hawking, 1977)

$$V_\theta^D \equiv \frac{\mu^\epsilon}{2\mathcal{A}} \mathrm{Tr}\, \ln\left(\frac{P_\theta(D)}{\mu^2}\right) = -\frac{\mu^\epsilon}{2\mathcal{A}} \lim_{s\to 0} \partial_s \zeta_\theta(s, D), \tag{3.11}$$

where

$$\zeta_\theta(s, D) = \mathrm{Tr}\left(\frac{P_\theta(D)}{\mu^2}\right)^{-s} = \frac{2\mu^{2s}}{\Gamma(s)} \int_0^\infty \frac{d\xi}{\xi} \xi^{2s} \mathrm{Tr}\big[e^{-\xi^2 P_\theta(D)}\big]. \tag{3.12}$$

It should be noted that the operator P_θ is positive and therefore the integral is well behaved at large ξ.

In order to find out which is the pole divergence in the limit $D \to 5$, one introduces the asymptotic expansion of the trace for small ξ (DeWitt, 1975),

$$\mathrm{Tr}\left[e^{-\xi^2 P_\theta(D)}\right] \sim \sum_{n=0}^{\infty} \xi^{n-D} a_{n/2}^D(P_\theta), \tag{3.13}$$

where $a_{n/2}^D$ are the so-called Seeley–De Witt coefficients. For $n \leq 5$ their explicit form is known for a wide class of covariant operators, which includes our P_θ. They are finite and can be constructed from geometric invariants integrated over spacetime. Given some specified boundary conditions and the form of P_θ, the Seeley–De Witt coefficients can be worked out using the results found, e.g., in Bordag *et al.* (1996), Kirsten (1998, 2001), Moss and Dowker (1989), and Vassilevich (1995). The integral (3.12) is well behaved for large ξ. For small ξ, the integral is convergent for $2s > D$, as can be seen from the asymptotic expansion (3.13). In the end, we have to consider the limit $s \to 0$, and so we must keep track of divergences that may arise in this limit. For this purpose, it is convenient to separate the integral into a small ξ region, with $\xi < \Lambda$, and a large ξ region with $\xi > \Lambda$, where Λ is some arbitrary cutoff. Substituting (3.13) into (3.12), we can explicitly perform the integration in the small ξ region for $2s > D$. This gives

$$\zeta(s, D) \sim 2\frac{\mu^{2s}}{\Gamma(s)} \left\{ \sum_{n=0}^{\infty} \frac{\Lambda^{n-D+2s}}{n-D+2s} a_{n/2}^D(P_\theta) + \int_\Lambda^\infty \frac{d\xi}{\xi} \xi^{2s}\, \mathrm{Tr}\big[e^{-\xi^2 P_\theta(D)}\big] \right\}. \tag{3.14}$$

The second term in curly brackets is perfectly finite for all values of s. Analytically continuing and taking the derivative with respect to s at $s = 0$ we have

$$\zeta'(0, D) \sim \sum_{n=0}^{\infty} \frac{2\Lambda^{n-D}}{n-D} a^D_{n/2}(P_\theta) + \text{finite}, \tag{3.15}$$

where the last term is just twice the integral in (3.14) evaluated at $s = 0$. Introducing the regulator $\epsilon = 5 - D$, now we identify the ultraviolet divergent part of V_θ^D, given by

$$V_\theta^{\text{div}} = -\frac{1}{\epsilon \mathcal{A}} a^D_{5/2}(P_\theta). \tag{3.16}$$

The divergence is removed by renormalizing the couplings in front of the invariants that make up the coefficient $a^D_{5/2}$. The renormalized effective potential of our interest is therefore given by

$$V_\theta = \lim_{D\to 5} \left[V_\theta^D + \frac{1}{\epsilon \mathcal{A}} a^D_{5/2}(P_\theta) \right]. \tag{3.17}$$

To proceed, we need to calculate V_θ^D, which in principle requires calculating a trace that involves the eigenvalues of P_θ, and as mentioned above, these are not related in any simple way to the KK masses. However, it turns out that the dimensionally regularized V_θ^D is independent of θ when D is not an integer. To show that this is the case, let us consider the generalized asymptotic expansion (Branson and Gilkey, 1990; McKean and Singer, 1967),

$$\text{Tr}\left[f(x)\, e^{-\xi^2 P_\theta} \right] \sim \sum_0^{\infty} \xi^{n-D} a^D_{n/2}(f, P_\theta), \tag{3.18}$$

where we have introduced the generalized Seeley–De Witt coefficients $a^D_{n/2}(f, P_\theta)$, constructed from local geometric operators and from the covariant derivatives of the (smooth) test function f. Again, explicit expressions for them are known for $n \leq 5$, although they will not be necessary for the present discussion. Note also that $a^D_{n/2}(P_\theta) = a^D_{n/2}(f = 1, P_\theta)$. The dependence of V_θ^D on θ can be found using the proper time representation given by Eqs. (3.11) and (3.12). One finds (see Garriga *et al.*, 2001)

$$\partial_\theta \lim_{s\to 0} \partial_s \zeta_\theta(s, D) = \lim_{s\to 0} \partial_s \frac{2\mu^{2s}}{\Gamma(s)} \int_0^{\infty} d\xi\, \xi^{2s} \partial_\xi \text{Tr}\left[-f_\theta e^{-\xi^2 P_\theta} \right], \tag{3.19}$$

where $f_\theta \equiv \partial_\theta \ln \Omega_\theta$. One may again introduce the regulator Λ and separate the integral into a large ξ part with $\xi > \Lambda$, which is finite and a small ξ part with $\xi < \Lambda$ which contains the divergent ultraviolet behavior. Using (3.18), we can integrate by parts assuming that $2s > D$. The resulting integrals in the small ξ region can be performed explicitly and we have

$$\partial_\theta \lim_{s\to 0} \partial_s \zeta_\theta(s, D) \sim \lim_{s\to 0} \partial_s \frac{4s\mu^{2s}}{\Gamma(s)} \left[\sum_{n=0}^{\infty} \frac{\Lambda^{n-D+2s}}{n-D+2s} a^D_{n/2}(f_\theta, P_\theta) + \text{finite} \right]. \tag{3.20}$$

As before, the last term just indicates the integral in the large ξ region. Provided that D is not an integer, all terms in square brackets remain finite at small s, and so the right-hand side of (3.20) vanishes.

Hence, we find that

$$\partial_\theta V^D_\theta = 0, \quad (D \neq \text{integer}). \tag{3.21}$$

In particular, this means that the dimensionally regularized determinant of P_θ coincides with the dimensionally regularized determinant of P_0, and we have

$$V^D_\theta = V^D_0 \equiv V^D \equiv \sum_n \mu^\epsilon \frac{1}{2} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \log\left(\frac{k^2 + m_n^2(\varphi_i, D)}{\mu^2} \right), \quad (D \neq \text{integer}). \tag{3.22}$$

Finally, from (3.17) and (3.22), we can write the renormalized effective potential in a form which is ready for explicit evaluation,

$$V_\theta(\varphi_i) = \lim_{D\to 5} \left[V^D - \frac{1}{(D-5)} \frac{1}{\mathcal{A}}\, a^D_{5/2}(P_\theta) \right]. \tag{3.23}$$

The above equation bears the ambiguity in the choice of integration measure in the second term in square brackets. Different values of θ give different results. If we take g_{ab} as the preferred metric, then we should use $\theta = 1$, whereas if we take $g^{(s)}_{ab}$ as the preferred metric, we should use $\theta = -1/\beta$. It can be proven (see Garriga *et al.*, 2001) that when we set $D = 5$ the coefficient $a_{5/2}(P_\theta)$ is also independent of θ. Hence, the pole term in the second term in (3.23) is independent of θ, as it should, in order to cancel the pole in V^D. As we will explicitly see, it is the finite part of V^{div}_θ that depends on the choice of θ.

4. EXPLICIT EVALUATION

For simplicity we shall restrict attention to the case of massless fields with arbitrary coupling to the curvature:

$$E = -\xi \mathcal{R}_g,$$

and with Dirichlet boundary conditions $\chi(z_\pm) = 0$. Here we shall use the method of dimensional regularization.

The eigenmodes of P_0 are given by

$$\chi_0^{\{n,k\}} = e^{ik_\mu x^\mu} z^{1/2}(A_1 J_\nu(m_n z) + A_2 Y_\nu(m_n z)).$$

The index of the Bessel functions is given by

$$\nu(D) = \frac{1}{2}\sqrt{1 - 4(D-1)\beta[(D-2)\beta - 2](\xi - \xi_c(D))}, \tag{4.1}$$

where

$$\xi_c(D) = \frac{1}{4}\frac{D-2}{D-1},$$

is the conformal coupling in dimension D. Imposing the boundary conditions on both branes, we obtain the equation that defines implicitly the discrete spectrum of m_n,

$$F(\tilde{m}_n) = J_\nu(\tilde{m}_n\eta)Y_\nu(\tilde{m}_n) - Y_\nu(\tilde{m}_n\eta)J_\nu(\tilde{m}_n) = 0, \tag{4.2}$$

where we have defined

$$\tilde{m}_n = m_n z_-, \quad \eta = \frac{z_+}{z_-}. \tag{4.3}$$

In the last section we concluded that the renormalized expression for the effective potential is

$$V_\theta(\varphi_i) = \lim_{D\to 5}\left[V^D - \frac{1}{(D-5)}\frac{1}{\mathcal{A}}a^D_{5/2}(P_\theta)\right]. \tag{4.4}$$

Consider first V^D, given in Eq. (3.22). Performing the momentum integrations, we obtain

$$V^D = -\frac{1}{2(4\pi)^2}(4\pi\mu^2)^{\epsilon/2}\frac{1}{z_-^{4-\epsilon}}\Gamma(-2+\epsilon/2)\tilde{\zeta}(\epsilon - 4), \tag{4.5}$$

where

$$\tilde{\zeta}(s) = \sum_n \tilde{m}_n^{-s}. \tag{4.6}$$

This regularized expression for the effective potential is finite when the real part of ϵ is sufficiently large.

Now the problem reduces to the computation of $\tilde{\zeta}$, which can be done in the same way as in the case discussed in Garriga *et al.* (2000). Skipping the detailed derivation, we simply give the final result for the regularized potential V^D (Garriga *et al.*, 2001):

$$V^D = \frac{1}{(4\pi)^2}\left[\left\{\left(\frac{1}{\epsilon} + \frac{3}{4} - \frac{\gamma}{2} + \frac{1}{2}\ln\left(4\pi\mu^2 z_0^2\right)\right)\beta_4 + \beta_4'\right\}\left(\frac{1}{z_-^4} + \frac{1}{z_+^4}\right)\right. \tag{4.7}$$
$$+\beta_4\left(\frac{1}{z_-^4}\ln\left(\frac{z_-}{z_0}\right) + \frac{1}{z_+^4}\ln\left(\frac{z_+}{z_0}\right)\right) + \frac{\mathcal{I}_K}{z_<^4} + \frac{\mathcal{I}_I}{z_>^4}$$

$$+ \int_0^\infty dx\, x^3 \ln\left(1 - \frac{I_\nu(xz_<)}{I_\nu(xz_>)}\frac{K_\nu(xz_>)}{K_\nu(xz_<)}\right)\Bigg] + \mathcal{O}(\epsilon). \tag{4.8}$$

For a scalar field with Dirichlet boundary conditions

$$\beta_4 = \frac{1}{128}(13 - 56\nu^2 + 16\nu^4). \tag{4.9}$$

In order to express the result for both $q > 1$ and $q < 1$ cases simultaneously, we have introduced $z_>$ and $z_<$ as the largest and the smallest of z_+ and z_-, respectively. The constant coefficients $\mathcal{I}_K(\nu)$, $\mathcal{I}_I(\nu)$ are calculable in principle, although their precise value is perhaps not very interesting since as we shall see these coefficients can be reabsorbed by finite renormalization. In Eq. (4.8), β_4 and β_4' stand for the vaules of β_4 and $d\beta_4(D)/dD$ evaluated at $D = 5$, respectively.

The next step is to subtract the divergent contribution. For the class of conformally flat spacetimes considered here, the Seeley–De Witt coefficient $a^D_{5/2}(P_\theta)$ can be computed for a generic dimension D, so we can expand the second term in the r.h.s. of Eq. (3.23) as

$$\frac{1}{(D-5)}\frac{1}{\mathcal{A}}a^D_{5/2}(P_\theta) = \frac{1}{(4\pi)^2}\left[\frac{1}{\epsilon}\beta_4\left(\frac{1}{z_-^4} + \frac{1}{z_+^4}\right) - \beta\theta\ \beta_4\left(\frac{1}{z_-^4}\ln\left(\frac{z_-}{z_0}\right) + \frac{1}{z_+^4}\ln\left(\frac{z_+}{z_0}\right)\right) - \delta\left(\frac{1}{z_-^4} + \frac{1}{z_+^4}\right)\right], \tag{4.10}$$

where β is given in Eq. (2.11), and δ is a constant whose value is not important since it can be absorbed by redefinition of μ. We can see that the divergent parts of the two terms in Eq. (3.23) cancel.

After redefinition of μ and addition of finite counterterms proportional to the invariants present in $a_{5/2}$, we can write the finite result (Garriga *et al.*, 2001)

$$V_\theta(z_+, z_-) = \frac{\beta_4(\beta\theta + 1)}{(4\pi)^2}\left[\frac{\ln(\mu_1 z_+)}{z_+^4} + \frac{\ln(\mu_2 z_-)}{z_-^4}\right] + \frac{1}{(4\pi)^2}\int_0^\infty dx\, x^3 \ln\left[1 - \frac{I_\nu(xz_<)}{I_\nu(xz_>)}\frac{K_\nu(xz_>)}{K_\nu(xz_<)}\right], \tag{4.11}$$

where μ_1 and μ_2 are renormalization constants.

5. HIERARCHY GENERATION

As already mentioned, we are interested in the possibility that the potential (4.11) can stbilize the moduli. Especially attractive is the situation when the location of the minimum in moduli space corresponds to a large separation between the mass scale of fields living on the negative tension brane and the effective four-dimensional Planck mass. It is well known that with an exponential warp factor it is easy to generate a large hierarchy (Randall and Sundrum, 1999). In this section we review how such a hierarchy can appear in the model given by (2.1).

The effective four-dimensional Planck mass in our model is given by

$$m_p^2 = \frac{2}{1+2q} M^3 y_+ \left[1 - \left(\frac{y_-}{y_+} \right)^{2q+1} \right], \tag{5.1}$$

where M is the five-dimensional Planck mass (see, e.g., Eq. (2.13) and (2.14)), and, without loss of generality, we have taken $y_+ = y_0$. Here, and for the rest of this section, we shall refer all physical quantities to the measurements of clocks and rods located on the positive tension brane.

Let us now consider the mass scales of fields that live on the branes. We expect them to couple not only to the metric, but also to the background scalar field ϕ. There are many possible forms for this coupling, but here we shall only consider those which respect the scaling symmetry ((2.15) and (2.16)). Thus, for a free scalar field Ψ that lives on the negative tension brane, and whose mass parameter is comparable to the cutoff scale, we may have an action of the form

$$S_\Psi = -\frac{1}{2} \int \sqrt{g_-^{(s)}} F^2(\phi) \left[g_-^{(s)\mu\nu} \partial_\mu \Psi \partial_\nu \Psi + f M^2 \Psi^2 \right]. \tag{5.2}$$

Here we have introduced a fudge factor f to allow for an intrinsic mass that is slightly lower than the cutoff scale. The function $F(\phi)$ can be reabsorbed in a redefinition of Ψ, and thus the relevant warp that determines the hierarchy between mass scales on the positive and in the negative tension branes is the one corresponding to the metric $g^{(s)}$. Then, the field Ψ will be perceived from the positive tension brane as having a mass squared of order

$$m^2 \sim f M^2 \left(\frac{y_-}{y_+} \right)^{2q-2}. \tag{5.3}$$

Thus, there are two different agents that contribute to the hierarchy between m and m_p. One is the small warp factor $(y_-/y_+)^{q-1}$ appearing in Eq. (5.3), which "redshifts" the mass scales of particles on the negative tension brane (except for $q < 1$, in which case the particles on the negative tension brane appear to be heavier than those on the positive tension brane). This generates the hierarchy in the RS model. The other is the possibly large volume of the internal space, which may enhance the effective Planck scale with respect to the cutoff scale (see Eq. (5.1)). This generates the hierarchy in the ADD model (Antoniadis *et al.*, 1998; Arkani-Hamed *et al.*, 1998, 1999) with large extra dimensions. Considering both effects, the hierarchy h is given by

$$h^2 = \frac{m^2}{m_p^2} \sim f \, \frac{1+2q}{2} \frac{1}{M y_+} \left(\frac{y_-}{y_+} \right)^{2q-2}.$$

It is known that without a warp factor, it is not possible to generate the desired hierarchy from a single extra dimension, since its size would have to be astronomical. An interesting question is what is the minimum value of the exponent q that would be sufficient in order to generate a ratio $m/m_p \sim 10^{-16}$. The best we can do is to take the curvature scale (y_+/q) slightly below the millimeter scale,

$$(y_+/q) \lesssim m_p (\text{TeV})^{-2} \sim \text{mm},$$

in order to pass the short distance tests on deviations from Newton's law, and

$$(y_-/q) \gtrsim M^{-1},$$

since for smaller values of y_- the curvature becomes comparable to the cutoff scale M and the theory cannot be trusted. Substituting in (5.1) we have $M^3 \gtrsim m_p (\text{TeV})^2$ and $(y_-/y_+) \gtrsim (m_p/\text{TeV})^{-4/3}$, which leads to

$$10^{-32} \sim \frac{m^2}{m_p^2} \gtrsim f \left(\frac{\text{TeV}}{m_p}\right)^{\frac{4(2q-1)}{3}}. \tag{5.4}$$

Hence, a warp factor with exponent $q \geq 5/4$ may account for the observed hierarchy with a single extra dimension, but it appears that this cannot be done for lower values of q.[4] In particular, the Heterotic M-theory model, with $q = 1/6$, does not seem to allow for such possibility.

6. STABILIZATION OF MODULI

In general, the effective potential induced by massless bulk fields with arbitrary curvature coupling is given by (4.11). In the limit when the branes are very close to each other, it behaves like (Garriga *et al.*, 2001) $V \propto a^4 |y_+ - y_-|^{-4}$, which is of the form of the potential induced by a conformally coupled scalar (Garriga *et al.*, 2001). It corresponds to the usual Casimir interaction in flat space. Perhaps more interesting is the moduli dependence due to local operators induced on the branes, which are the dominant terms in $V(y_+, y_-)$ when the branes are widely separated. Such operators break the scaling symmetry of the classical action discussed in Section 2, but nevertheless are needed in order to cancel the divergences in the effective potential.

In this limit of large interbrane separation, the potential (4.11) assumes a "Coleman–Weinberg" form for each one of the moduli (Garriga *et al.*, 2001),

$$V(y_+, y_-) \approx \sum_{i=\pm} a^4(y_i) \left\{ \alpha K^4(y_i) \ln \left[\frac{K(y_i)}{\mu_i} \right] + \delta\sigma_i \right\}. \tag{6.1}$$

[4] Except, of course, by giving up the assumption that the Lagrangian of matter on the branes should scale in the same way as the rest of the classical action (see the discussion around Eq. (5.2)). If we allow any coupling of ϕ to the mass term for Φ, then any hierarchy can be easily generated for any value of q.

Here, we have introduced the "curvature scale" $K(y) = q/y$, so that $K^4(y_i)$ behaves like a generic geometric operator of dimension 4 on the brane (such as the fourth power of the extrinsic curvature). The constant α in (6.1) is given by

$$\alpha = \frac{(1-\theta)q - 1}{(4\pi)^2} (1 - q^{-1})^4 \sum \beta_4^{(\chi)}, \tag{6.2}$$

where we sum over the contributions from all bulk fields χ. The numerical coefficients $\beta_4^{(\chi)}$ are given by Eqs. (4.9) and (4.1).

The value of θ depends on the choice of integration measure in the path integral that defines the effective potential (see Section 3). If we adopt the point of view that this measure should be covariant with respect to the Einstein frame metric g_{ab} that enters our original action functional (2.1), then we should take $\theta = 1$. However, this is not the only possible choice. The classical action has a scaling symmetry that transforms both g_{ab} and the background scalar field ϕ. Using a conformal transformation that involves the scalar field, we may construct a new metric $g_{ab}^{(s)}$ that does not transform under scaling. If we require that the path integral measure should be covariant with respect to this new metric, then we should take $\theta = 1 - 1/q$. With this particular choice of θ the coefficient α vanishes and the logarithmic terms in (6.1) disappear. This seems to indicate that this is a preferred choice for the measure, since in that case the local terms proportional to a^4K^4, which break the scaling symmetry, can be eliminated (at least at the one-loop level). Nevertheless, it is far from clear that this is indeed a preferred choice. Here we take the attitude that the parameter θ is unknown, and that it should be fixed by a more fundamental theory of which (2.1) is just a low energy limit.

The renormalization constants μ_i can be estimated by looking at the "renormalized coefficient" of the geometric terms of dimension 4 on the brane $c_i(K) = \alpha \ln(K/\mu_i)$. In the absence of fine-tuning, the $c_i(K)$ are expected to be of order one near the cutoff scale $K \sim M$, where M^{-3} is basically the five-dimensional Newton's constant. Hence, we expect

$$\mu_i \sim M\, e^{-c_i/\alpha}, \tag{6.3}$$

where $c_i = c_i(M) \sim 1$. In (6.1), we have also allowed for finite renormalization of local operators on each one of the branes. These operators are collectively denoted by $\delta\sigma_i$. In order to ensure that the effective potential V does not severely distort the background solution, this correction to the brane tension must be much smaller than the effective tension of the brane in the classical background solution. From the Darmois–Israel matching conditions, this effective tension is of order M^3K_i. Hence we require

$$\delta\sigma_i \ll M^3 K_i \ll M^4. \tag{6.4}$$

In Section 3 we considered contributions to the effective potential from massless bulk fields. These may have an arbitrary coupling to the curvature scalar of the standard form $\xi\mathcal{R}\chi^2$, or certain couplings to the background dilaton, such as the couplings occurring in the Heterotic M-theory (see Garriga *et al.*, 2001). However, if the model contains massive bulk fields, of mass m, then we expect terms proportional to m^2K^2 in the effective potential. Even without massive bulk fields, we may expect the presence of lower dimensional worldsheet operators of the form M^3K, M^2K^2, and MK^3, due to cubic, quadratic, and linear divergences in the effective theory. Hence, we may expect that $\delta\sigma_i$ has an expansion of the form

$$\delta\sigma_i(K_i) \sim \Lambda_i^4 + \gamma_{1i}M^3K_i + \gamma_{2i}M^2K_i^2 + \gamma_{3i}MK_i^3 + \mathcal{O}\big(K_i^5\big), \qquad (6.5)$$

where $K_i \ll M$, $\Lambda_i \ll M$, and $\gamma_{1i} \ll 1$ in order to satisfy (6.4).

The logarithmic terms may in principle stabilize the moduli at convenient locations. Note that this effect is due to the warp factor and vanishes in flat space (where the coefficients β_4 vanish). The effect also vanishes accidentally in the RS case, because the curvature scale $K(y)$ is constant. The position of the minima are determined by $\partial_{y_i}V = 0$. This leads to the conditions

$$\delta\sigma_i = \frac{\alpha}{q}K_i^4\left[(1-q)\ln\left(\frac{K_i}{\mu_i}\right) + \frac{1}{4}\right] + K_i\frac{\delta\sigma_i'}{4q}, \qquad (6.6)$$

where the prime on $\delta\sigma_i$ indicates derivative with respect to K_i. Also, we must require that the minima occur at an acceptable value of the effective cosmological constant. Using the condition (6.6), we can write the value of the potential at the minimum as

$$V_{\min} = \frac{K_+^4}{4q}\sum_{i=\pm}\left(\frac{K_i}{K_+}\right)^{4(1-q)}\left\{4\alpha\ln\left(\frac{K_i}{\mu_i}\right) + \alpha + K_i^{-3}\delta\sigma_i'\right\} \lesssim 10^{-122}m_p^4. \quad (6.7)$$

The latter condition will require one fine-tuning amongst the parameters in (6.5).

An interesting question is whether the effective potential (6.1) can generate a large hierarchy and at the same time give sizeable masses to the moduli. As discussed in Section 5, the hierarchy is given by

$$h^2 = \frac{m^2}{m_p^2} \sim \frac{K_+}{M}\left(\frac{K_+}{K_-}\right)^{2q-2}, \qquad (6.8)$$

where $m \sim$ TeV is the mass of the particles that live on the negative tension brane, as perceived by the observers on the positive tension brane. Consistency with Newton's law at short distances requires $K_+ \gtrsim (\mathrm{TeV})^2/m_p \sim (\mathrm{mm})^{-1}$, and consistency of perturbative analysis requires $K_- \lesssim M$. With these constraints, the observed hierarchy $h \sim \exp(-37)$ can only be accomodated for $q \gtrsim 5/4$. To proceed, we shall distinguish two different cases.

Case a. This is the generic case, where the coefficients γ_{1i}, γ_{2i}, and γ_{3i} in the expansion of $\delta\sigma_i(K)$ are not too suppressed. In this case, the logarithmic terms in the effective potential are in fact subdominant, and the minima of the effective potential are determined by $4q\delta\sigma_i \approx K_i\delta\sigma_i'$. Quite generically, this will lead to stabilization of the moduli near (or slightly below) the cutoff scale $K_i = \lambda_i M$, with $\lambda_i \sim 1$. Hence we have

$$h^2 \sim \exp[2(q-1)\ln(\lambda_+/\lambda_-)].$$

Since the logarithm is of order one, an acceptable hierarchy can be generated provided that $q \gtrsim 10$. This is "close" to the RS limit $q \to \infty$. In this case, $M \sim m_p$ and the size of the extra dimension is of order m_p^{-1}. On the positive tension brane the parameter Λ_+ has to be fine tuned so that the effective cosmological constant is 122 orders of magnitude smaller than the Planck scale. A straightforward calculation shows that the physical mass eigenvalues for the moduli $\varphi_\pm$ in the present case are given by

$$m_+^2 \sim q^{-2}m_p^{-2}K_+^4 \lesssim m_p^2, \quad m_-^2 \sim q^{-1}h^2 m_p^{-2}K_-^4 \lesssim h^2 m_p^2.$$

Thus, the lightest radion has a mass comparable to the TeV scale.

Case b. This corresponds to the case where almost all of the operators in (6.5) are either extremely suppressed or completely absent, due perhaps to some symmetry. In particular, we shall concentrate on the possibility that

$$\delta\sigma_i = \gamma_{1i} M^3 K_i,$$

since an operator of this form is already present in the classical action (2.1), and it is the only one in the expansion (6.5), which is allowed by the scaling symmetry. In this case, and assuming for simplicity that the negative tension brane is near the cutoff scale $K_- \sim M$, we can rewrite (6.7) as

$$V_{\min} \sim \frac{3\alpha K_+^4}{(4q-1)}\left\{\left(\ln(K_+/\mu_+) + \frac{1}{3}\right) + h^{8(q-1)/(2q-1)}\left(\ln(K_-/\mu_-) + \frac{1}{3}\right)\right\}.$$

For $q > 1$, the first term dominates and the condition of a nearly vanishing cosmological constant forces $K_+ \approx \mu_+ e^{-1/3}$. A fine-tuning of Λ_+ will be necessary in order to satisfy the condition (6.6) for such value of K_+. The hierarchy is given by

$$h^2 \sim \left(\frac{\mu_+}{M}\right)^{2q-1} \sim \exp[-(2q-1)\alpha^{-1}c_+], \tag{6.9}$$

where μ_+ is given by (6.3). Since the effective coupling α can be rather small, a large hierarchy may be obtained even for moderate $q \gtrsim 1$. A straightforward calculation shows that at the minima of the effective potential (6.1) $\partial^2_{\varphi_+} V = 12\alpha(1+2q)^{-2}a_+^4K_+^4\varphi_+^{-2}$, and $\partial^2_{\varphi_-} V \sim \alpha q^{-1}a_-^4K_-^4\varphi_-^{-2}$. Hence, we find that the physical masses for the moduli fields φ_+ and φ_- that appear in (2.13) are given by

$$m_+^2 \sim \alpha q^{-2} h^{12/(2q-1)}\, m_p^2, \quad m_-^2 \sim \alpha q^{-1} h^{2+4/(2q-1)} m_p^2.$$

Associated with the eigenvalue m_+ there is a Brans–Dicke (BD) field,[5] with BD parameter $\omega_{\rm BD} = -3q/(1+2q)$. Therefore, we must require $m_+ \gtrsim (\text{mm})^{-1}$, which in turn requires $q > 2$. A stronger constraint on q comes from the eigenvalue m_-, since the corresponding field is coupled to ordinary matter with TeV strength. The mass of this field cannot be too far below the TeV, otherwise it would have been seen in accelerators. This requires q to be rather large $q \gtrsim 10$. So, we see from Eq. (6.8) that in order to get the observed value for the hierarchy h respecting this bound forces the parameter α to be almost of order one, which corresponds to a large number of fields. Moreover, we find that the cutoff scale is $M \sim h^{1/(2q-1)} m_p$. So taking into account this bound, the cutoff is again of the order of m_p. Similarly we see that the size of the extra dimension is again of order m_p^{-1}.

7. CONCLUSIONS

We have calculated the one-loop effective potential for the moduli in a class of warped brane-world compactifications with a power-law warp factor of the form $a(y) = (y/y_0)^q$. Here y is the proper distance in the extra dimension. In general, there are two different moduli $y_\pm$ corresponding to the location of the branes (in the RS limit, $q \to \infty$, a combination of these moduli becomes pure gauge). We have presented the calculation in dimensional regularization, formalizing and extending the approach adopted in Flachi and Toms (2001), Goldberger and Rothstein (2000), and Toms (2000). An important point is that the divergent term to be subtracted from the dimensionally regularized effective potential is proportional to the the Seeley–De Witt coefficient $a_{5/2}$. In the RS model, this coefficient behaves much like a renormalization of the brane tension, but it behaves very differently in the general case.

The result provides a stabilization mechanism that can be summarized as follows. The scaling symmetry of the action (2.1) is responsible for the masslessness of the moduli at the tree level. The effective potential induced by a conformal scalar field in the bulk is finite because the conformal anomaly vanishes for the background spacetime topology. For that reason the potential fails to stabilize the moduli without fine-tunings if a large hierarchy is required. Instead, a nonminimally non-conformally coupled scalar, generates a divergent potential and local operators are inevitably generated on the branes. These operators break the scaling symmetry of the classical action and, subject to suitable renormalization conditions, stabilize the moduli. For an exponential warp factor (the Randall–Sundrum model), this does not apply because all these operators are constant. For a sufficiently steep

[5] Here we are considering the situation where the mass of φ_- is much larger than the mass of φ_+, and where the visible matter is on the negative tension brane. In this case, since $y_- = \text{const.}$, visible matter is universally coupled to the metric $g^{(-)}_{\mu\nu}$, and the BD parameter corresponding to φ_+ can be read off from (2.13).

warp factor, $q \gtrsim 10$, only one renormalization constant needs to be fine-tuned in order to obtain a large hierarchy, siezable values for the masses of the moduli and a sufficiently small cosmological constant. This feature is in common with the Goldberger and Wise (1999) mechanism for the stabilization of the radion in the RS model. For $q \lesssim 10$, the stabilization is also possible, but if we also demand that the hierarchy $h \sim 10^{-16}$ is generated geometrically, then the resulting masses for the moduli would be too low. In conclusion, it seems that the anomalous breaking of the scaling symmetry of the action[6] is what lies behind this stabilization mechanism.

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